Financial Intermediation and Credit Market Equilibrium: A Model of Matching Market

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Abstract

We analyse an incentive model of financial market where intermediaries with different monitoring technologies are matched with firms with different levels of initial wealth and a project. Firms do not have sufficient wealth to cover the project costs and hence, seek external financing. The intermediaries are the potential investors in the market. We model the financial economy as a two-sided matching game and analyse the equilibrium using stability as a solution concept. In equilibrium, the financial contracts are optimal, and payoffs consumed by firms and intermediaries are endogenous. We also show that, in equilibrium, poorer firms have to rely on more informed capital available in the market and suffer from more intensive monitoring.

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1 Introduction

A large set of literature on financial markets conclude that when external borrowing by firms is indispensable, capital-poor firms are denied credits by uninformed investors and they have to rely on informed capital available in the economy.[1] Informed capital owns, in general, better monitoring technology compared to less informed investors. Hence, it is better able to cope with *moral hazard* at the firm level that arises because of the inability to contract upon all the actions taken by the entrepreneurs in a firm seeking credit. This moral hazard problem is more severe with the poorer firms, and hence they fail to obtain credit from the uninformed investors. Poorer firms are sometimes able to invest by borrowing from informed capital which is supplied by financial intermediaries but only after being monitored more intensively.

The main goal of this paper is to analyse a financial economy consisting of firms (run by entrepreneurs) with different levels of start-up capital, and financial intermediaries (or, investors) with different monitoring technologies. In this financial market, several firms and intermediaries interact with each other which calls for a general equilibrium framework. In this model, the payoff of each individual is determined endogenously, unlike the standard financial market models where the payoffs are determined exogenously. The framework also allows us to establish the identities of the intermediaries who become potential sources of credit to different firms.

We model the financial market as a two-sided *matching game*. If a firm convinces an intermediary to finance his project, we say that the firm and the intermediary are *matched* to form a pair. A matching is a rule that specifies all such possible pairs in the economy. An outcome of this market is an endogenous matching and a set of financial contracts, one for each firm-intermediary pair under the matching. A financial contract specifies that the intermediary finances the project and receives state-contingent claims on the project return. Each firm operates on his project after he obtains external finance and chooses a non-contractible effort level. Choice of effort influences the probability of having a high return from the project. Firm's liability is limited to his current income. Hence, differences in wealth imply differences in liabilities. We use *stability* as the equilibrium concept. An outcome is stable if there is no intermediary-firm pair that would be (strictly) better-off than under the initial outcome.

We characterise the equilibrium of the financial market. First, all contracts are Pareto optimal, i.e., given the others being equally well-off as before, no individual can strictly improve upon his/her situation in the outcome by signing a different contract. Second, there is always a subset of the firms (the poorest ones) which fail to obtain credit from any external source. If the firms form the long-side of the market, the size of this (unmatched) set is even larger. Also, capital-rich firms earn higher payoffs. On the other hand, if the intermediaries form the long-side of the market then less informed intermediaries stay out of business earning zero profit.

[1] See Hölmstrom and Tirole [8], and Repullo and Suárez [10].

The equilibrium payoff of each individual is endogenous in this model. We are able to establish bounds on the payoff of each firm, and these bounds depend on the other pairs formed in the economy.

Next, in this financial market with both-sided heterogeneity, we show that in a stable outcome the matching is negatively assortative, i.e., capital-poor firms rely on more informed capital, and they are monitored more intensively. In this framework, for an intermediary-firm pair, firm capital and monitoring intensities turn out to be substitutes in producing as well as transferring the surplus between each other. Negatively assortative matching patterns are consequences of this two-fold substitutability.

Our model bears resemblance with the financial intermediation models proposed by Hölmstrom and Tirole [8], and Repullo and Suárez [10]. In both papers the authors consider models of bank monitoring under moral hazard. In these models, the financial economies are characterised by a continuum of firms with different wealth levels, and small number of investors. There are two types of investors: insiders and outsiders. The outside investors are not able to monitor the firms as intensively as the insiders can. In the model of Hölmstrom and Tirole [8], the investors are capital constrained, and they face a moral hazard problem at the level of monitoring. They show that capital-rich firms prefer outsiders to raise finance, whereas capital-poor firms have to rely on bank (insiders) finance to invest in their projects. They also analyse the effects of different types of capital tightening in the economy. Any sort of these adverse shocks hits the poor firms more severely by taking them out of business and leads to a contraction in the bank loan which is only geared towards rich firms thus implying, what Bernanke *et. al.* [4] call, a *flight to quality*. A change in the intermediary capital also affects the monitoring levels. Repullo and Suárez [10] consider similar kind of model where the intermediaries are not capital constrained. They draw similar conclusions as the previous paper. Moreover, they find an intermediate range of firm's wealth where the firms can choose between outside finance and bank credit. Moreover, they also study the effects of interest rate spread on the equilibrium. Besanko and Kanatas [5] also consider a theoretical model of endogenous bank monitoring and show that firms might suffer from excessive monitoring in equilibrium.

In this paper we consider a discrete set of firms and intermediaries. This allows us to deal with small as well as large number of individuals. In the papers cited above, although a general (competitive) equilibrium framework is considered, intermediary payoff is determined by its reservation value and hence, the intermediaries of each type break-even. Since an endogenous matching market is used in the current model, the payoff each individual earns is determined endogenously. The equilibrium matching pattern, namely negatively assortative matching, conforms to the findings of Hölmstrom and Tirole [8], and Repullo and Suárez [10], if we consider only two types of investors: insider with higher monitoring intensity and outsiders with lower monitoring intensity. We later consider a many-to-one market where an investor can finance more than one project only with the restriction that the project returns are uncorrelated. Another difference between the model of Hölmstrom and Tirole [8] and that of ours is that they consider both the intermediaries and firms are capital constrained, whereas in the current model only firms lack capital to fund their project. In this sense, we only have demand-side considerations.

Our model is built on the theoretical model proposed by Dam and Pérez-Castrillo [7], where the authors characterise the set of stable outcomes of a principal-agent economy with identical principals and heterogenous agents. From matching theory point of view, this paper is also related to works by Becker [3], Crawford and Knoer [6], Legros and Newman [9], and Serfes [11], which, as Dam and Pérez-Castrillo [7] do, feature no or imperfect transferability of surplus among the agents. This non-transferability is typical to agency models characterised by provision of incentives since optimality is not implied by maximisation of total surplus. Becker [3], and Legros and Newman [9], in a more general matching framework, also provide sufficient conditions for positively and negatively assortative matching patterns in equilibrium.

In a recent empirical paper, Ackerberg and Botticini [1], using a data set on agricultural contracts between landlords and tenants in early Renaissance Tuscany, show that, although the characteristics of a land owned by landlord is exogenous, the kind of tenant attracted to it is determined by an *endogenous matching*. They further asset

> *...principals with more ability to monitor or more ability to measure output (who might relatively prefer low share or high royalty contracts) might end up matching with agents with more risk-averion, more credit constraints, or a higher cost of effort (who might also relatively prefer low share or wage contracts).*

The above suggests that strong influence of endogenous matching is not unusual in determining the terms of incentive compatible contracts among individuals.

The paper is organised as follows. In Section 2, we lay out the basic model and describe the optimal financial contracts. We define matching and outcome for the market in the following section. We describe the main results in Section 3. In the next section, we consider the case when intermediaries are able to finance more than one firms. We conclude in Section 5.

2 The Financial Economy

2.1 Firms and Intermediaries

We consider an economy with a *finite* set of n risk neutral *firms*, who own a project of (fixed) size 1 apiece. A generic firm is identified by his level of initial wealth (or, start-up capital) w^j. We arrange the firms according to their wealth levels in descending order as $1 > w^1 \geq w^2 \geq$

.... $\geq w^n \geq 0$. Firm's initial wealth is not sufficient to cover the entire project cost, hence each firm seeks external finance. There are m risk-neutral intermediaries (or, banks) with different monitoring technologies identified by the monitoring intensities. These intermediaries are the potential investors in the market. A firm has to convince an intermediary to finance his project. Intermediary i is identified by her monitoring intensity v_i. We arrange the intermediaries with respect to their monitoring intensities in descending order as $v_1 \geq v_2 \geq .... \geq v_m$. Intermediaries with higher monitoring intensity are often referred to as *more informed capital*. An intermediary with intensity v_i incurs a fixed cost v_i and commit to monitor a firm she finances. Clearly, intermediary v_1 owns the best monitoring technology and intermediary v_m owns the worst one. We assume that the monitoring technology does not permit an intermediary to control more than one firm.[2] Also each intermediary incurs per unit opportunity cost of fund which is equal to ρ. Intermediaries and firms are matched in pairs. We allow for the possibility that a firm can seek for an alternative financier. Hence, the matching is endogenous rather than being exogenous. Whenever matched, an intermediary-firm pair signs a financial contract and the intermediary finances the entire project.[3] Firm's wealth works as a collateral in the project even though, the firm does not invest his wealth in the project.

When an intermediary agrees to finance a project, the firm undertakes an effort $p \in [0, 1]$ which influences the idiosyncratic states *Success* or *Failure*. Each project yields a return y in the event of success which happens with probability p, and 0, in case of failure with complementary probability. The firm can divert the finance it obtains to private uses in order to gain private benefit Φ by choosing a lower probability of success. The intermediary can make the firm she finances behave more diligently by monitoring his project. Hence, private benefit decreases with both the probability of success, and the monitoring intensity. We consider private benefit of the form $\Phi(p, v_i) = \frac{1-p^2}{2v_i}$.[4] The above form of private benefit function implies that higher monitoring intensity reduces the entrepreneurial private benefits obtained under a given probability of success. Notice that for each p, expected return from a project decreases with monitoring intensity, and hence, firms, given a choice, prefer lower intensity of monitoring.

2.2 Financial Contracts

An intermediary-firm pair (v_i, w^j) signs a contract $c = (R, r)$ that specifies state-contingent claims, $R > 0$, in case of success and $r > 0$, in the event of failure.[5] The choice of p is not

[2] In section 4, we will comment on the case where each intermediary finances more than one project.

[3] There is no loss of generality if we assume that the entire project is financed by the intermediary since, firm's wealth is observable.

[4] Our main results hold good for any function that is decreasing and concave in p and decreasing in v_i.

[5] For notational convenience, we suppress subscripts and superscripts to identify a contract. But (R, r) will be different for each intermediary-firm pair.

contractible. Let p maximise the firm's utility

$$p \in \underset{p'}{argmax}\ \{p'(y-R) - (1-p')r + \frac{1-p'^2}{2v_i}\}. \tag{IC}$$

This is the *incentive compatibility* constraint. The expected utilities of intermediary v_i and firm w^j when they sign the contract c will be:

$$\begin{aligned} u_i(w^j, c) &= pR + (1-p)r, \\ u^j(v_i, c) &= p(y-R) - (1-p)r + \frac{1-p^2}{2v_i}. \end{aligned}$$

In the above, we consider intermediary's utility *gross* of her cost of capital, and firm's utility *net* of his wealth. We denote by c^{null}, the *null contract*, under which all individuals consume zero utility. Suppose, firm w^j earns 0 if he does not get finance from any intermediary. Firm's and intermediary's *individual rationality constraints* are given by:

$$\begin{aligned} pR + (1-p)r - \rho(1+v_i) &\geq 0, \\ p(y-R) - (1-p)r + \frac{1-p^2}{2v_i} &\geq 0. \end{aligned}$$

Firm's liability is limited to the state-contingent return plus his initial wealth. *Limited liability* implies

$$R \leq y + w^j\ , \tag{LS}$$

$$r \leq w^j\ . \tag{LF}$$

The assumption of risk neutrality together with limited liability makes the incentive compatibility constraint costly and hence, it gives rise to *moral hazard* at the firm level. The incentive compatibility constraint implies that the firm may choose any probability in [0, 1]. Since firm's utility is (strictly) concave in p, one can replace (IC) by the *first order condition* of the firm's maximisation problem as follows:

$$p = v_i(y - R + r). \tag{IC$'$}$$

We also make the following assumptions.

ASSUMPTION 1 *$v_i \in [\frac{1}{y}, 1]$ for all v_i.*

A contract for an intermediary-firm pair must satisfy the individual rationality and limited liability constraints. We club all these natural restrictions into the following definition.[6]

DEFINITION **1 (Feasibility)**
A contract is **feasible for a firm** *w^j if it satisfies the restrictions of individual rationality and limited liability.*

[6]Notice that limited liability constraints are agent specific.

Denote by $\mathcal{X}^j$ the set of contracts that are feasible for firm w^j. From now on we will concentrate only on feasible contracts. Let $\mathcal{Z}^j$ be the subset of feasible contracts that are incentive compatible and consider the following programme.

$$\begin{cases} \max_{c \in \mathcal{Z}^j} u_i(w^j,\, c) \\ s.t. \quad u^j(v_i,\, c) \geq \bar{u}^j, \end{cases} \tag{P}$$

where the utility levels are given by $u_i(w^j,\, c)$ and $u^j(v_i,\, c)$, taking into account that the probability of success is given by the incentive compatibility constraint (IC$'$). Let $c_i(\bar{u}^j)$ be the set of contracts that solve the above maximisation programme. This is a *classic* example of the set of contracts often discussed in the principal-agent literature when only a *fixed* principal-agent relationship is concerned. In other words, this is the set of feasible and incentive compatible contracts when one abstract from a *principal-agent market*, and only focus on a particular (given) relationship. The payoff to the firm is determined entirely by the individual rationality constraint, and hence exogenous. As we have mentioned earlier that one of the main goals of this paper is to endogenise the payoff of the firm, in the following sections, we concentrate on a market where many intermediaries and firms interact, and consequently the payoff of each firm is determined endogenously and influenced by the other intermediary-firm pairs formed in the economy.

The contract $c_i(\bar{u}^j)$ generates the utility possibility frontier for a pair $(v_i,\, w^j)$. We represent the frontier by $u(v_i,\, w^j,\, \bar{u}^j)$, which is the maximum payoff of intermediary v_i when firm w^j receives a payoff $\bar{u}^j$. As regard to the above programme (P), notice that the utility of intermediary v_i is maximised given that firm w^j obtains a minimum amount $\bar{u}^j$. Hence, the set of contract that solve the above programme is the set of Pareto optimal contracts. A contract $c_i(0)$ is a point on utility possibility frontier where the firm gets zero payoff. Obviously, the frontier corresponds to the set of Pareto optimal contracts.

We now describe the contract $c_i(\bar{u}^j)$ for the pair $(v_i,\, w^j)$.[7] In order to do that, first consider the optimal contract under symmetric information, when the incentive compatibility and the limited liability constraints do not have any bite. In other words, the firm does not face any moral hazard. It is easy to check that the intermediary v_i offers a contract which makes sure that firm w^j chooses $p = 1$ and obtains zero private benefit. Also, the firm makes a fixed transfer equal to $y - \bar{u}^j$ to the intermediary. Below we summarise the optimal contract under moral hazard, i.e., the situation where the combination of risk-neutrality and limited liability makes the incentive compatibility typically costly for the intermediary. In this situation for a sufficiently high level of firm's wealth the limited liability constraint does not bind, and the optimal contract corresponds to the one under symmetric information. This is known as the *first-best* contract. For a lower level of firm's wealth, the contract is subject to moral hazard

[7]See Appendix A for detailed analysis of the optimal contracts.

and the first-best is not achieved. Following is the description of the optimal contracts under moral hazard. The state contingent payments are determined as follows.

$$r = \begin{cases} y - \bar{u}^j, & \text{if } \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j < w^j, \\ w^j, & \text{if } \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j \geq w^j. \end{cases}$$

and

$$R = \begin{cases} y - \bar{u}^j, & \text{if } \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j < w^j, \\ y + w^j - \frac{1}{v_i}\sqrt{2v_i(w^j + \bar{u}^j) - 1}, & \text{if } \frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j \leq w^j \leq \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j, \\ \frac{y}{2} + w^j, & \text{if } \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j > w^j. \end{cases}$$

Notice that one can identify three disjoint regions over which the above contracts are candidates for optimum. First, when $\frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j < w^j$, firm's participation constraint binds, but the limited liability (in the event of failure) does not. Here the contract is *first best*. In the other two regions, limited liability constraint binds and the moral hazard problem becomes costly for the intermediary. An optimal contract under moral hazard typically lies in the region $\frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j \geq w^j$. Within this region, for a very low level of firm's wealth ($\frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j > w^j$), provision of incentive implies that the firm gets payoff strictly higher than $\bar{u}^j$. The probability of success is given by:

$$p = \begin{cases} 1, & \text{if } \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j < w^j, \\ \sqrt{2v_i(w^j + \bar{u}^j) - 1}, & \text{if } \frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j \leq w^j \leq \frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j, \\ \frac{v_i y}{2}, & \text{if } \frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j > w^j. \end{cases}$$

If one looks at the payoff of an intermediary v_i, then it is clear that she would be willing to finance a project owned by a firm w^j only if $w^j \geq \tilde{w}$,[8] Suppose there are n_0 firms in the economy with wealth level below $\tilde{w}$. These n_0 firms will always be denied credit and stay out of the market. Also notice that, if the opportunity cost of fund, ρ, increases then the number of unfunded projects (n_o) also increases.

The above equations show that the p chosen by the firm is always strictly positive, and that the probability of success is continuous function of $\bar{u}^j$. In the following lemma we show that, under moral hazard, any intermediary prefers to finance a firm with higher wealth and lower payoff.

LEMMA 1 *Under moral hazard, if $w^j > w^{j'}$ and $\bar{u}^j \leq \bar{u}^{j'}$, then $u(v_i, w^j, \bar{u}^j) > u(v_i, w^j, \bar{u}^{j'})$ for any v_i.*

PROOF See Appendix B. ■

[8] Where $\tilde{w}$ is given by $\tilde{w} \equiv \rho(1 + v_i) - \frac{vy^2}{4}$. See Appendix A for details.

2.3 Matching

When an intermediary agrees to finance a project an intermediary-firm pair is formed, and a contract is signed. The following three definitions describe a matching and a relevant outcome for this financial market. Let us denote the set of intermediaries and the set of firms by $\mathcal{I} = \{v_1, v_2, ..., v_m\}$ and $\mathcal{F} = \{w^1, w^2, ..., w^n\}$, respectively.

Definition **2** **(Matching)**
A (one-to-one) **matching** *for the market is a mapping* $\mu : \mathcal{I} \cup \mathcal{F} \to \mathcal{I} \cup \mathcal{F}$ *such that (i)* $\mu(v_i) \in \mathcal{F} \cup \{v_i\}$ *for all* $v_i \in \mathcal{I}$, *(ii)* $\mu(w^j) \in \mathcal{I} \cup \{w^j\}$ *for all* $w^j \in \mathcal{F}$ *and (iii)* $\mu(w^j) = v_i$ *if and only if* $\mu(v_i) = w^j$ *for all* $(v_i, w^j) \in \mathcal{I} \times \mathcal{F}$.

The above definition implies that a matching for the market is a mapping which specifies that either each individual of one side of the market is assigned to another individual of the other side, or the individual stays alone. We say that the pair (v_i, w^j) is *matched* under μ if $\mu(v_i) = w^j$ (or equivalently, $\mu(w^j) = v_i$).

Definition **3** *A menu of contracts* $\mathcal{C}$ **compatible with a matching** μ *for the market is a vector of feasible contracts,* $\mathcal{C}$ *one for each pair matched according to* μ.

Definition **4** **(Outcome)**
An **outcome** $(\mu, \mathcal{C})$ *for the market is a matching* μ *and a menu of contracts compatible with the matching.*

The outcomes of the market we describe here are endogenous. This endogeneity has two aspects. First, the contracts signed by the intermediaries and the firms are endogenous. The second aspect is that the matching itself should be endogenous. In the following section we determine the set of *equilibrium* financial contracts for the market as well as which firms are financed by which intermediaries. We require that a *reasonable* outcome should be immune to the possibility of being *blocked* by any intermediary-firm pair (as well as by any single individual). In other words, an outcome for a market has to be *stable*. Denote by c_i^j the contract signed by $(v_i \, w^j)$.

Definition **5** **(Stability)**
An outcome $(\mu, \mathcal{C})$ *for the market is* **stable** *if there does not exist any pair* (v_i, w^j) *and any contract* $c' \in \mathcal{X}^j$ *such that* $u_i(w^j, c') > u_i(\mu(v_i), c_i^{\mu(v_i)})$ *and* $u^j(v_i, c') > u^j(\mu(w^j), c^j_{\mu(w^j)})$.

The above definition implies that no intermediary-firm pair with a feasible contract can *block* the current outcome. The restriction of individual rationality implies that no individual firm or intermediary unilaterally blocks the outcome.

3 The Set of Stable Outcomes

In this section we describe the set of stable outcomes of the market. We start by stating a couple of important properties of a stable outcome.

First, all the contracts in a stable outcome are optimal. By optimality we mean that there is no possibility of (strictly) improving the payoff of any individual in an intermediary-firm pair without making the other individual worse-off. The following lemma states this optimality property.

LEMMA **2** *All the contracts in a stable outcome are optimal.*

PROOF Suppose $(\mu, \mathcal{C})$ is stable, but the contract $c_i^j \in \mathcal{C}$ signed by v_i and w^j, where $\mu(w^j) = v_i$, is not optimal. Then there exists a contract c', feasible for (v_i, w^j) such that (i) $u_i(w^j, c') > u_i(w^j, c_i^j)$ and (ii) $u^j(v_i, c') > u^j(v_i, c_i^j)$. In that case (v_i, w^j) will block $(\mu, \mathcal{C})$ with c'. This contradicts the fact that $(\mu, \mathcal{C})$ is initially stable. ■

It is worth noting that the optimality of a contract between an intermediary and a firm in any stable outcome is guaranteed by the possibility that the same pair can block the initial outcome with a different feasible contract. Hence, a contract signed by a matched pair (v_i, w^j) must maximise the expected utility of one party taking into account that the other gets at least a certain utility level. One particular class of optimal contracts are discussed in Section 2.2. The optimality of the contracts in a stable outcome implies that the set of contracts that solve the programme (P) form a part of the stable outcome where firm w^j consumes $\bar{u}^j$. This makes sure that if in a stable outcome if a firm w^j, being matched with intermediary v_i, receives $\bar{u}^j$, then the contract between (v_i, w^j) is $c_i^j = c_i(\bar{u}^j)$ and the payoff to intermediary v_i is $u(v_i, w^j, \bar{u}^j)$.

Another property of stable outcome is that among the matched ones, a richer firm is always better-off, which is shown in the following lemma.

LEMMA **3** *In a stable outcome for the market, for any w^j and $w^{j'}$ matched, $\bar{u}^j > \bar{u}^{j'}$ if $w^j > w^{j'}$. And for any w^j and $w^{j'}$ unmatched, $\bar{u}^j = \bar{u}^{j'} = 0$.*

PROOF Suppose $w^j > w^{i'}$ are both matched and in a stable outcome $\bar{u}^{j'} \geq \bar{u}^j$. By Lemma 1, $u(\mu(w^{j'}), w^j, \bar{u}^j) > u(\mu(w^{j'}), w^{j'}, \bar{u}^{j'})$. Hence, there exists $c' = c_{\mu(w^{j'})}(\bar{u}^j) - \varepsilon$ for $\varepsilon > 0$, small enough such that (i) $u_{\mu(w^{j'})}(w^j, c') = u(\mu(w^{j'}), w^j, \bar{u}^j) - \varepsilon > u(\mu(w^{j'}), w^{j'}, \bar{u}^{j'})$ and $u^j(\mu(w^{j'}), c') \geq \bar{u}^j + \varepsilon > \bar{u}^j$.[9] Hence, $(\mu(w^{j'}), w^j)$ would block the outcome with c', which contradicts the stability of the initial outcome.

[9]For any contract, $c - \varepsilon = (R - \varepsilon, r - \varepsilon)$.

The second part follows directly from the fact that if a firm is unmatched then he signs a contract c^{null}. Also note that if a firm w^j is matched, then by individual rationality $\bar{u}^j \geq 0$. ■

In the following definition we introduce the concept of *willingness to pay*, which will be used in characterising the set of stable outcomes.

DEFINITION **6** **(Willingness to Pay)**
Given any outcome $(\mu, \mathcal{C})$ where firms w^j and $w^{j'}$ obtain $\bar{u}^j$ and $\bar{u}^{j'}$, respectively, for any intermediary v_i the willingness to pay for w^j against $w^{j'}$ is defined as

$$\Delta u_i(j, j') \equiv u(v_i, w^j, \bar{u}^j) - u(v_i, w^{j'}, \bar{u}^{j'}).$$

This expression means that for given levels of initial wealth, w^j and $w^{j'}$, and payoffs, $\bar{u}^j$ and $\bar{u}^{j'}$, if intermediary v_i is currently engaged with $w^{j'}$, the above quantity is the maximum additional amount she is willing to pay to contract with w^j instead, or this is the maximum extra amount she will pay to keep w^j in case v_i is currently with w^j rather than any other firm $w^{j'}$.[10]

The above two lemmas provide only a partial characterisation of the set of stable outcomes. Let $n_0 = n - n'$ be the number of firms who are always denied credit. In the following theorem we characterise the set of stable outcomes.

THEOREM **1** *An outcome $(\mu, \mathcal{C})$ for the financial market is stable if and only if the following conditions hold:*

(a) *Only as many $\gamma \equiv min\{m, n'\}$ pairs are formed. If the numbers of firms and intermediaries are different, then either the subset $\{w^{m+1}, w^{m+2}, ..., w^{n'}\}$ of firms, or the subset $\{v_{n'+1}, v_{n'+2}, ..., v_m\}$ of intermediaries remains unmatched,*
(b) *The contracts signed are $c_i(\bar{u}^{\mu(v_i)})$ if v_i is matched, and c^{null} for any unmatched individual.*
(c) *If $w^j > w^{j'}$, then $\Delta u_{\mu(w^j)}(j, j') \geq \bar{u}^j - \bar{u}^{j'} \geq \Delta u_{\mu(w^{j'})}(j, j')$ for all w^j, $w^{j'}$ matched. And $\bar{u}^j = 0$ for all w^j unmatched.*

PROOF We first prove that **(a)**-**(c)** are necessary conditions for an outcome $(\mu, \mathcal{C})$ to be stable.

(a) It is easy to see that a maximum of γ pairs are formed, since the matching game is one-to-one. Suppose that in a stable outcome strictly less than $\gamma = m$ pairs are formed. Then there must be at least one intermediary, say v_i and one firm, say w^j are unmatched, earing zero payoffs. Then there exists a contract $c' = c_i(0) - \varepsilon$ such that $u_i(w^j, c') = u(v_i, w^j, 0) - \varepsilon > 0$ and $u^j(v_i, c') \geq \varepsilon > 0$. Hence, (v_i, w^j) blocks the outcome with c', which is a contradiction.

[10]Notice that $\Delta u_i(j, j')$ depends on v_i, w^j, $w^{j'}$, $\bar{u}^j$ and $\bar{u}^{j'}$.

The above implies that if there are same number of firms and intermediaries, then there is no firm or no intermediary remains unmatched.

Suppose there are more firms than intermediaries, and in a stable outcome a firm w^j with $j < m$ is unmatched. This firm gets zero payoff. If w^j is unmatched, then there must be some w^k with $k \geq m$ matched. Given Lemma 3, $u^k(\mu(w^k), c)$ must be equal to zero since $w^j > w^k$. Then following Lemma 1, there exists a contract $c' = c_{\mu(w^k)}(0) - \varepsilon$ such that $u_{\mu(w^k)}(w^j, c') = u(\mu(w^k), w^j, 0) - \varepsilon > u(\mu(w^k), w^k, 0)$ and $u^j(\mu(w^k), c') \geq \varepsilon > 0$. Thus, the pair $(\mu(w^k), w^j)$ blocks the outcome with c', which is a contradiction.

Now suppose that there are more intermediaries than firms in the financial market and in a stable outcome an intermediary v_i with $i < n$ is unmatched. Then there is some v_k with $k \geq n$ is matched with some firm, say w^j. It is easy to check that $u(v_i, w^j, \bar{u}^j) > u(v_k, w^j, \bar{u}^j)$ since $v_i > v_k$.[11] Then there exists a contract $c' = c_i(\bar{u}^j) - \varepsilon$ with which the pair (v_i, w^j) blocks the outcome.

(b) Follows from Lemma 2.

(c) We show that in a stable outcome $(\mu, \mathcal{C})$, one cannot have (i) $\Delta u_{\mu(w^{j'})}(j, j') > \bar{u}^j - \bar{u}^{j'}$, and (ii) $\bar{u}^j - \bar{u}^{j'} > \Delta u_{\mu(w^j)}(j, j')$. In (i), notice that the term on the *left hand side* of the inequality is the willingness to pay of intermediary $\mu(w^{j'})$ for w^j against $w^{j'}$, and the *right hand side* is the difference between the utilities obtained by w^j and $w^{j'}$, the extra (in terms of utilities) $\mu(w^{j'})$ has to pay if she would have offered the contract $c_{\mu(w^{j'})}(\bar{u}^j)$ to w^j instead of offering $c_{\mu(w^{j'})}(\bar{u}^{j'})$ to $w^{j'}$. This implies that $\mu(w^{j'})$ has an incentive to form a blocking pair together with w^j. And such a blocking pair is viable since there exists a contract $c' = c^{\mu(w^{j'})}(\bar{u}^j) - \varepsilon$ such that (a) $u_{\mu(w^{j'})}(w^j, c') = u(\mu(w^{j'}), w^j, \bar{u}^j) - \varepsilon > u(\mu(w^{j'}), w^{j'}, \bar{u}^{j'})$ and (b) $u^j(\mu(w^{j'}), c') \geq \bar{u}^j + \varepsilon > \bar{u}^j$. This contradicts the supposition that the outcome was initially stable.

For the other part, write (ii) as $\bar{u}^{j'} - \bar{u}^j < -\Delta u_{\mu(w^j)}(j, j') = \Delta u_{\mu(w^j)}(j', j)$. This expression is similar to (i). Hence, it is easy to check that with the contract $c' = c^{\mu(w^j)}(\bar{u}^{j'}) - \varepsilon$ intermediary $\mu(w^j)$ and intermediary $w^{j'}$ form a blocking pair, which is a contradiction.

The fact that any unmatched individual gets zero payoff trivially follows from **(b)**.

For sufficiency, we only need to prove **(b)** and **(c)**, which together with **(a)** imply stability. The inequalities in **(c)** imply

$$u(\mu(w^j), w^j, \bar{u}^j) - u(\mu(w^j), w^{j'}, \bar{u}^{j'}) \geq u(\mu(w^{j'}), w^j, \bar{u}^j) - u(\mu(w^{j'}), w^{j'}, \bar{u}^{j'})$$

[11] Consider the value function $u(v_i, w^j, \bar{u}^j)$ of programme (P′) in Appendix A. Then $\frac{\partial u}{\partial v} = \frac{y(w^j + \bar{u}^j)}{(2v_i(w^j+\bar{u}^j)-1)^{\frac{1}{2}}} - \frac{1}{v_i^2}$. Given that $v_i y > 1$, the above expression is strictly positive. It is obvious to check that in the other two regions also $u(v_i, ., .)$ is strictly increasing in v_i.

It follows from the above expression:

$$\bar{u}^j+u(\mu(w^j),\, w^j,\, \bar{u}^j)+\bar{u}^{j'}+u(\mu(w^{j'}),\, w^{j'},\, \bar{u}^{j'}) \geq \bar{u}^j+u(\mu(w^{j'}),\, w^j,\, \bar{u}^j)+\bar{u}^{j'}+u(\mu(w^j),\, w^{j'},\, \bar{u}^{j'})$$

The last inequality implies that the matching μ is optimal in the sense that no other configuration such as $(\mu(w^j),\, w^{j'})$ can make any individual strictly better-off. Thus, the optimality of both the contracts $c_i(\bar{u}^j)$ and the matching μ implies there do not exist any contract c' different from $c_i(\bar{u}^j)$ and any pair who can block the outcome with c', and hence, the outcome is stable. ■

In the above theorem we characterise the set of stable outcomes of the financial market. First important property is that all financial contracts in a stable outcome are Pareto optimal. This is already established in Lemma 2. Consequently the financial contract to be signed by each intermediary-firm pair is the contract that solves the maximisation problem (P). Then we show that a richer firm earns higher payoff in a stable outcome. This property is robust irrespective of the matching pattern in a stable outcome. Obviously, had the firms been equal with respect to their initial wealth, they would have consumed same payoffs.

We are also able to establish a lower bound and an upper bound to the payoff a firm obtains in a stable outcome. Following Lemma 3, a richer firm earns higher payoff. Part (c) of the above theorem shows that the maximum payoff a firm w^j can get is the payoff obtained by $w^{j'}$, a poorer firm, plus the maximum extra amount (in terms of payoff) that his matched partner is willing to pay to be matched with him rather than being matched with $w^{j'}$. On the other hand, the minimum he can get is $\bar{u}^{j'}$ plus the maximum extra amount the intermediary he is matched with is willing to pay for being matched with him rather than with w^j. A straightforward interpretation of this is that the intermediaries compete for the right to be matched with richer firms. And also an intermediary matched with a richer firm clearly out bids her counterpart.

In Section 2.2 we have asserted that a firm being matched with an intermediary is equivalent to the firm is not denied credit. The intermediaries prefer to finance an economically viable projects rather than investing capital in the outside market and earning a risk-free interest rate. One can notice from part (a) of the above theorem that when firms form the long-side of the market, given the one-to-one game, only m firms get their projects financed. Also, only the richest firms and all the intermediaries are matched and the poorest firms stay out of business. More interestingly, when the intermediaries form the long-side of the market, only the best monitors stay in the market.

In the following theorem we show that the set of stable outcomes of the financial market is non-empty.

THEOREM **2** *For the financial market described above, the set of stable outcomes is non-empty.*

PROOF   We omit a technical proof. In the discussion that follows the theorem, we relate the matching game described in Section 2.3 to the matching games analysed in Alkan and Gale [2], and Crawford and Knoer [6]. Then it is immediate to prove the existence of a stable outcome for the financial market studied here. ■

In the current model, we describe a one-to-one matching game with individuals engaged in trading units of contracts. Our framework can be seen as a generalisation of the *assignment game* between buyers and sellers described by Shapley and Shubik [12]. In the assignment game, buyers, who own single indivisible objects apiece, are matched with sellers. In this case, an outcome is a matching and a vector of prices for each buyer-seller pair under the matching. In our model, the transactions occur via contracts rather than prices. The analysis of the optimal contract signed by an intermediary-firm pair immediately reveals, as is typical in the incentive models, that the optimality is not implied by the maximisation of total surplus of a particular pair. In other words, the downward-sloping Pareto frontier $u(v_i, w^j, \bar{u}^j)$ of a pair (v_i, w^j) is non-linear in the utility space. From the maximisation problem described earlier (resulting from a non-cooperative game played between an intermediary-firm pair) one can see that this non-linearity implies imperfect transferability of surplus between the contracting parties. The matching game described in Section 2.3 is an equivalent cooperative game played between firms and intermediaries. Our game, unlike the assignment game of Shapley and Shubik [12], is characterised by imperfect transferability of surplus. Given the set of individuals, $\mathcal{I} \cup \mathcal{F}$, and the Pareto frontier for a matched pair (v_i, w^j), the matching game described above is similar to a general game described by Alkan and Gale [2], and the game between firms and workers described by Crawford and Knoer [6]. A proof of the above theorem, i.e., the set of outcomes is non-empty, is same as the proofs provided in the above mentioned papers, and hence we omit it in order to avoid technicalities. Observe that, the assignment game of Shapley and Shubik [12] is a special case of ours since the Pareto frontier in their game can be written as $u(v_i, w^j, \bar{u}^j) \equiv f(v_i, u^j) - \bar{u}^j$, where intermediary v_i can transfer surplus perfectly to firm w^j.

Next we analyse the matching pattern in a stable outcome. We would look at cases when the characteristic of an individual is monotone in the characteristic of his/her matched partner, i.e., the stable matching is *monotone*. Two types of monotone matching can be considered. First, if richer firms are matched with more efficient monitors, then we say that the matching is *positively assortative* (PAM). On the other hand, if richer firms are monitored less intensively, then the matching is *negatively assortative* (NAM). The following definition formalises these concepts.

DEFINITION **7** *Given a matching μ under which firms w^j and $w^{j'}$ are matched, μ is negatively (positively) assortative if $w^j > w^{j'}$ implies $\mu(w^j) < (>)\ \mu(w^{j'})$.*

A negatively assortative matching implies that poorer firms obtain finance from intermediaries with higher monitoring intensities. On the other hand, in a positively assortative matching richer firms are monitored more intensively. In the following theorem we show that in a stable outcome, we can always guarantee negatively assortative matching pattern. This kind of matching occurs if an intermediary with lower monitoring intensity out bids an intermediary with higher monitoring level if both compete for being matched with a richer firm. This is the case when the willingness to pay for a richer firm is decreasing in monitoring intensity.

THEOREM **3** *In a stable outcome matching is negatively assortative.*

PROOF First we show that in a stable outcome if the willingness to pay decreases then the matching is negatively assortative. Consider the following *decreasing willingness to pay* condition.

$$\Delta u_i(j, j') \leq \Delta u_{i'}(j, j') \quad \text{for} \quad v_i > v_{i'} \quad \text{and} \quad w^j > w^{j'}. \tag{DWP}$$

Then the above and Part (c) of Theorem 1 together imply that in a stable outcome we must have $\mu(w^j) = v_{i'}$ and $\mu(w^{j'}) = v_i$, and hence μ is negatively assortative.

The only thing remains to be shown is that, given stability, the condition (DWP) is always satisfied. As we discuss in Section 2.3 that the solution to programme (P) is candidate to be optimal over three disjoint regions of the parameter space. It is easy check that under first-best and when the firm's individual rationality constraint is not binding $\Delta u_i(j, j') = \Delta u_{i'}(j, j')$ for $v_i > v_{i'}$ and $w^j > w^{j'}$.[12] So (DWP) is automatically satisfied.

To see this in the intermediate region, consider the Pareto frontier $u(v_i, w^j, \bar{u}^j)$. It can be proven that $\frac{\partial^2 u}{\partial w^j \partial v_i} \leq 0$ and $\frac{\partial^2 u}{\partial \bar{u}^j \partial v_i} \leq 0$. Take $w^j > w^{j'}$ and $v_i > v_{i'}$. Also, in this case, in a stable outcome $\bar{u}^j < \bar{u}^{j'}$ since, $w^j > w^{j'}$. This implies:

$$\begin{aligned} u(v_i, w^j, \bar{u}^j) - u(v_{i'}, w^j, \bar{u}_i) &\leq u(v_i, w^{j'}, \bar{u}^j) - u(v_i, w^{j'}, \bar{u}^j) \quad (1) \\ u(v_i, w^{j'}, \bar{u}^j) - u(v_{i'}, w^{j'}, \bar{u}^j) &\leq u(v_i, w^{j'}, \bar{u}^{j'}) - u(v_{i'}, w^{j'}, \bar{u}^{j'}). \quad (2) \end{aligned}$$

The above two together imply

$$u(v_i, w^j, \bar{u}^j) - u(v_i, w^{j'}, \bar{u}^{j'}) \leq u(v_{i'}, w^j, \bar{u}^j) - u(v_{i'}, w^{j'}, \bar{u}^{j'})$$

This is nothing but (DWP), and hence the theorem. ■

In the above theorem we have shown that, in equilibrium, the poor firms can manage to get fund only after being monitored more severely. As the set of Pareto optimal contracts in a stable outcome is the set of contracts that solve the programme (P), negative assortment of matching is guaranteed under moral hazard as well as under the *first-best*. This implies that the poorer

[12] See Appendix C for a detailed proof of the theorem.

firms have to rely on more informed capital in order to finance their projects. More informed intermediaries monitor more intensively. When the intermediaries are more in number, quite naturally in equilibrium, only the better monitors survive in the market.

The above result conforms to the findings of Hölmstrom and Tirole [8], and Repullo and Suárez [10]. In the next section, we allow for intermediaries to finance more projects with the restrictions that project returns are uncorrelated.

In this model, as in the two papers mentioned above, negatively assortative matching is a consequence of substitutability between firms and intermediaries. When contracts involve provision of incentives, for a given intermediary-firm pair optimality is not equivalent to total surplus maximisation. The total surplus generated in a relation crucially depends on the way the surplus is divided. In other words, a relationship is constrained by imperfect transferability of surplus between the contracting parties. Hence, substitutability has two aspects in this model. First, monitoring intensity and wealth level are substitutes in producing the surplus. Secondly, they are also substitutes in transferring the surplus from one party to another. The condition of decreasing willingness to pay is nothing but restating this substitutability property which guarantees a match between poorer firms and excessive monitoring in equilibrium.

4 An Intermediary Finances More Firms

Previously we have assumed that each intermediary finances only one firm. In this section, instead, we consider that intermediary v_i can finance a maximum of q_i (quota of v_i) firms. Furthermore, for simplicity, we assume that there is no cost of fund, i.e., $\rho = 0$.[13] Also there are m intermediaries and n firms, where $n > m$ and $\sum_{i=1}^{m} q_i = n$. Assume further that the project returns are not correlated. Notice that when an intermediary can finance more than one project, the market becomes a *many-to-one* replica of the one-to-one model described previously. The formal definition of this type of matching is given below.

Definition **8 (Many-to-one Matching)**
A (many-to-one) **matching** *for the market is a mapping* $\mu : \mathcal{I} \cup \mathcal{F} \to \mathcal{I} \cup 2^{\mathcal{F}}$ *such that (i)* $\mu(v_i) \in 2^{\mathcal{F}}$ *for all* $v_i \in \mathcal{I}$*, (ii)* $\mu(w^j) \in \mathcal{I} \cup \{w^j\}$ *for all* $w^j \in \mathcal{F}$ *and (iii)* $\mu(w^j) = v_i$ *if and only if* $w^j \in \mu(v_i)$ *for all* $(v_i, w^j) \in \mathcal{I} \times \mathcal{F}$.

It is straightforward to extend the results obtained in the previous section. Consider an intermediary v_i with quota q_i. This can be replicated as q_i identical intermediaries with monitoring intensity v_i. Let this set be $\mathcal{I}_i = \{v_{i_1}, v_{i_2}, ..., v_{i_{q_i}}\}$ and $\mathcal{I} = \mathcal{I}_1 \cup .... \cup \mathcal{I}_m$. Hence, we have a one-to-one model with n intermediaries and n firms. We can order the intermediaries according

[13]In this section, we will not provide formal proofs as they follow trivially from the previous ones.

to their costs as $v_{1_1} = ... = v_{1_{q_1}} > v_{2_1} = ... = v_{2_{q_2}} > .... > v_{m_1} = ... = v_{m_{q_m}}$. We characterise the set of stable outcome in the following proposition.

PROPOSITION 1 *An outcome* $(\mu, \mathcal{C})$ *is stable for the many-to-one market if and only if the following conditions hold:*

(a) *Each intermediary* v_i *finances* q_i *firms and all* n *firms are financed,*
(b) $c_i^j = c_i(\bar{u}^j)$ *for any* $v_i \in \mathcal{I}$,
(d) $\Delta u_{\mu(w^j)}(j, j') \geq \bar{u}^j - \bar{u}^{j'} \geq \Delta u_{\mu(w^{ij'})}(j, j')$ *for all* $i, i' \in \{1, 2, ..., m\}$.

Also in the stable outcome the matching is negatively assortative, i.e., a poorer firm is monitored more severely by his financier.

The main body of the paper focuses on a one-to-one model. The modification to a many-to-one model is nothing but a replica of the one-to-one market as we assume away any correlation among the projects. Had the projects been correlated, then the contracts solving (P) would not always been optimal. Extending our model to the case of correlated projects would then call for a more sophisticated contract design which is beyond the scope of the current model.

A special case of the model in this section is to consider only two intermediaries with quotas q_1 and q_2 such that $q_1 + q_2 = n$ and $v_1 > v_2$. Then a consequence of stability is that the poorer q_1 firms will be financed by v_1 and the richer $q_2 = m - q_1$ firms will rely on v_2 to finance their project. This also occurs in Repullo and Suárez [10]. The intermediary v_1 is then called *informed capital* and v_2, the *uninformed capital.* In this model we do not allow for the possibility that more than one intermediary can invest in one firm. Hölmstrom and Tirole [8] provides two interpretations of monitoring. First, allowing for the above possibility, monitoring serves as certification of firm's solvency that attracts uninformed capital into the firm. Second, when only one investor finances a project, monitoring is interpreted as intermediation that helps firms, which are denied credit by uninformed investors, invest in their projects by raising informed capital. The current model takes the second strand, and hence, throughout the paper we refer to the investors as *intermediaries.*

5 Conclusion

In this paper we model a financial economy as two-sided matching game and characterise the set of stable outcomes. We show that when firms need to raise external fund to finance their projects, in equilibrium, the capital-poor firms have to rely on more informed capital in the market, and they suffer from excessive monitoring. This conforms to the findings of Hölmstrom and Tirole [8], and Repullo and Suárez [10]. Unlike these two works, ours is a model with finite number of individuals and we do not allow for any correlation among the project returns. But the use of matching games to model the financial market allows us endogenise the payoffs of all the participating individuals. We also propose a very simple framework that is able to solve general (competitive) equilibrium models of financial markets characterised by incentive problems. The payoffs of richer firms are typically higher in equilibrium. It is worth noting that, the results obtained in Theorems 1 are robust to any equilibrium matching patterns.

The current model leaves several avenues for future research. We consider a one-to-one matching game with several intermediaries and several firms, which is later extended to a many-to-one model with the restriction of no correlation among the project returns. Incorporation of this feature would call for more sophisticated contract design and a many-to-one matching model, which would not be a trivial extension of the financial market described in the current paper. The equilibrium in the case with correlated projects would then facilitate to analyse the effects of different kinds of macroeconomic shocks as done by Hölmstrom and Tirole [8], and Repullo and Suárez [10]. Another extension would be to allow more than one investor to invest in the same firm. In this case, as Hölmstrom and Tirole [8] interpret, if a more informed investor monitors the firm then it works as certifying the firm's solvency and helps attract external capital into the firm from less informed investors. Finally, one can extend the model by making the intermediaries capital constrained. This would give rise to a moral hazard problem in the level of monitoring, and it would also be more rational then to consider correlated project returns in a many-to-one setup.

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Appendix

A. Analysis of the Optimal Contract

We solve for the optimal contract for an intermediary-firm pair $(v_i,\, w^j)$. The contract should solve the following maximisation programme:

$$\max_{\{p,\,R,\,r\}} u_i \equiv pR + (1-p)r \tag{P}$$

$$\begin{aligned} \text{s.t.} \quad u_i \equiv p(y-R) - (1-p)r + \frac{1-p^2}{2v_i} && (3)\\ v_i(y-R+r) = p && (4)\\ R \le y + w^j && (5)\\ r \le w^j. && (6) \end{aligned}$$

Since, constraint (4) is satisfied with equality we can substitute for R in the objective function and the other constraints in order to obtain the following reduced programme:

$$\max_{\{p,\,r\}} py - \frac{p^2}{v_i} + r \tag{P$'$}$$

$$\begin{aligned} \text{s.t.} \quad \frac{p^2}{2v_i} - r + \frac{1}{2v_i} \ge \bar{u}^j && (7)\\ r \le w^j. && (8) \end{aligned}$$

Let ν_1 and ν_2 be the Lagrange multipliers of the above programme. The Kuhn-Tucker (first-order) conditions are given by:

$$\begin{aligned} y - \frac{2p}{v_i} + \nu_1 \frac{p}{v_i} &= 0 && (9)\\ 1 - \nu_1 - \nu_2 &= 0 && (10)\\ \nu_1\left(\frac{p^2}{2v_i} - r + \frac{1}{2v_i} - \bar{u}^j\right) &= 0 && (11)\\ \nu_2(w^j - r) &= 0 && (12)\\ \frac{p^2}{2v_i} - r + \frac{1}{2v_i} - \bar{u}^j &\ge 0 && (13)\\ w^j - r &\ge 0 && (14)\\ \nu_1,\, \nu_2 &\ge 0. && (15) \end{aligned}$$

We consider the following cases.

Case 1: $\nu_1 = \nu_2 = 0$. This is not compatible with equation (10).

Case 2: $\nu_1 > 0$ and $\nu_2 = 0$. Let $(p^*,\, R^*,\, r^*)$ be the candidate solution in this case. From (10), $\nu_1 = 1$. Then from (9) one gets $p = v_i y$. Given $v_i y \ge 1$, $p^* = 1$. From constraint (4) in programme (P) and equation (11), one gets

$$R^* = r^* = y - \bar{u}^j.$$

The utilities are given by:

$$\begin{aligned} u_i{}^* &= y - \bar{u}^j, \\ u^j{}^* &= \bar{u}^j. \end{aligned}$$

Finally, the solution must satisfy (14), i.e.,

$$\frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j < w^j.$$

Hence, for $(v_i,\, w^j)$ in the above region, $(p^*,\, R^*,\, r^*)$ is candidate for an optimum. In this region, the contract is the *first-best* contract where the provision of incentive does not involve any cost.

CASE 3: $\nu_1 = 0$ and $\nu_2 > 0$. Then from equation (12), $r^0 = w^j$. From equations (10) and (9) we have $p^0 = \frac{v_i y}{2}$. Then from constraint (4) of programme (P) we get $R^0 = \frac{y}{2} + w^j$. The utilities are given by:

$$\begin{aligned} u_i{}^0 &= \frac{v_i y^2}{4} + w^j, \\ u^j{}^* &= \frac{v_i y^2}{8} + \frac{1}{2v_i} - w^j. \end{aligned}$$

Notice that a firm w^j gets his project financed if $w^j \geq \tilde{w} \equiv \rho(1+v_i) - \frac{vy^2}{4}$, since for $w^j \geq \tilde{w}$, $u_i^0 \geq \rho(1+v_i)$. Finally, the solution must satisfy equation (13) which implies

$$\frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j > w^j.$$

Hence, for $(v_i,\, w^j)$ in the above region, $(p^0,\, R^0,\, r^0)$ is candidate for an optimum.

CASE 4: $\nu_1 > 0$ and $\nu_2 > 0$. Then from (12), $\widehat{r} = w^j$. Substituting this in equation (11) we get,

$$\widehat{p} = \sqrt{2v_i(w^j + \bar{u}^j) - 1}.$$

Then from constraint (4) of programme (P) we get

$$\widehat{R} = y + w^j - \frac{1}{v_i}\sqrt{2v_i(w^j + \bar{u}^j) - 1}.$$

The utilities are given by:

$$\begin{aligned} \widehat{u}_i &= y\sqrt{2v_i(w^j + \bar{u}^j) - 1} - 2\bar{u}^j - w^j + \frac{1}{v_i}, \\ \widehat{u}^j &= \bar{u}^j. \end{aligned}$$

Since, $\nu_1 > 0$ from equation (9) we have $v_i y - 2\widehat{p} \leq 0$. This implies

$$\frac{v_i y^2}{8} + \frac{1}{2v_i} - \bar{u}^j \leq w^j$$

Also $\nu_1 < 1$ implies that $v_i y - \widehat{p} \geq 0$ (equation (9)). Hence we get

$$\frac{v_i y^2}{2} + \frac{1}{2v_i} - \bar{u}^j \geq w^j$$

Hence, for $(v_i,\, w^j)$ in the above region, $(\widehat{p},\, \widehat{R},\, \widehat{r})$ is candidate for an optimum. Given the previous analysis, we can conclude that the optimal contracts are those described in the text. ■

B. Proof of Lemma 1

Consider the value function $u(v_i,\, w^j,\, \bar{u}^j)$ of programme (P). The Lagrange function is given by: Using Envelope Theorem we get,

$$\frac{\partial u(v_i,\, w^j,\, \bar{u}^j)}{\partial w^j} = \nu_2 > 0.$$

The above implies:

$$u(v_i,\, w^j,\, \bar{u}^j) > u(v_i,\, w^{j'},\, \bar{u}^j) \quad \text{if} \quad w^j > w^{j'} \tag{a}$$

Also

$$\frac{\partial u(w^j,\, v_i,\, \bar{u}_i)}{\partial \bar{u}_i} = -\nu_1 < 0,$$

since, at the (incentive constrained) optimum $\nu_1 > 0$. Hence, we have

$$u(v_i,\, w^{j'},\, \bar{u}^j) > u(v_i,\, w^{j'},\, \bar{u}^{j'}) \quad \text{if} \quad \bar{u}^j < \bar{u}^{j'} \tag{b}$$

The above two together imply:

$$u(v_i,\, w^j,\, \bar{u}^j) > u(v_i,\, w^{j'},\, \bar{u}^{j'}) \quad \text{if} \quad w^j > w^{j'} \quad \text{and} \quad \bar{u}^j \leq \bar{u}^{j'}.$$

This completes the proof of the lemma. ■

C. Proof of Theorem 3

First we show that in a stable outcome if the willingness to pay decreases (condition (DWP)) then the matching is negatively assortative. Consider the following condition.

$$\Delta u_i(j,\, j') \leq \Delta u_{i'}(j,\, j') \quad \text{for} \quad v_i > v_{i'} \quad \text{and} \quad w^j > w^{j'}. \tag{DWP}$$

Then the above and Part (c) of Theorem 1 together imply that in a stable outcome we must have $\mu(w^j) = v_{i'}$ and $\mu(w^{j'}) = v_i$, and hence μ is negatively assortative.

The only thing remains to be shown is that, given stability, the condition (DWP) is always satisfied. As we have discussed earlier that the solution to programme (P) is candidate to be optimal over three disjoint regions of the parameter space. It is easy check that under first-best and when the firm's individual rationality constraint is not binding $\Delta u_i(j,\, j') = \Delta u_{i'}(j,\, j')$ for $v_i > v_{i'}$ and $w^j > w^{j'}$. So (DWP) is automatically satisfied.

To see this in the intermediate region, consider the maximum value function $u(v_i,\, w^j,\, \bar{u}^j)$ of the maximisation programme (P). From this we get

$$\frac{\partial^2 u}{\partial w^j \partial v_i} = \frac{\partial^2 u}{\partial \bar{u}^j \partial v_i} = y\sqrt{2v_i(w^j + \bar{u}^j) - 1}\left[1 - v_i(w^j + \bar{u}^j)(2v_i(w^j + \bar{u}^j) - 1)^{-1}\right] \leq 0,$$

since $\widehat{p} \leq 1$. The above equation implies:

$$u(v_i,\, w^j,\, \bar{u}^j) - u(v_{i'},\, w^j,\, \bar{u}_i) \leq u(v_i,\, w^{j'},\, \bar{u}^j) - u(v_i,\, w^{j'},\, \bar{u}^j) \tag{16}$$

$$u(v_i,\, w^{j'},\, \bar{u}^j) - u(v_{i'},\, w^{j'},\, \bar{u}^j) \leq u(v_i,\, w^{j'},\, \bar{u}^{j'}) - u(v_{i'},\, w^{j'},\, \bar{u}^{j'}). \tag{17}$$

The above two together imply

$$u(v_i,\, w^j,\, \bar{u}^j) - u(v_i,\, w^{j'},\, \bar{u}^{j'}) \leq u(v_{i'},\, w^j,\, \bar{u}^j) - u(v_{i'},\, w^{j'},\, \bar{u}^{j'})$$

This is nothing but (DWP), and hence the theorem. ■